QUESTIONS AND DISCUSSIONS.

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Three questions which, among others, have been standing for some time without having been answered are republished below in the hope that some of our readers may thereby be stimulated to send in suitable replies.

15. In the *Proceedings of the Royal Society of Edinburgh*, Vol. VII, p. 144, in some mathematical notes by Professor P. G. Tait, it is stated:

"If $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3y^3 + (x^3 - y^3)^3z^3 = (z^3 + y^3)^3x^3$.

"This furnishes an easy proof of the impossibility of finding two integers the sum of whose cubes is a cube."

How does this "easy proof" follow?

(A partial reply to the above has been received showing that if $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3y^3 + (x^3 - y^3)^3z^3 = (z^3 + y^3)^3x^3$. Can some one show how the "easy proof" then follows?)

21. For the diophantine equation

$$x^2 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{lrrrrrrrr} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 8, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given diophantine equation? How may all the solutions of this equation be found by a systematic procedure?

32. In a discussion of the Peaucellier[1] Cell by analytic methods the following equations are obtained:

$$\begin{array}{llll} (1) & (x_2 - x_1)^2 + (y_2 - y_1)^2 - b^2 = 0; & (2) & (x_3 - x_1)^2 + (y_3 - y_1)^2 - b^2 = 0; \\ (3) & (x_2 - X)^2 + (y_2 - Y)^2 - b^2 = 0; & (4) & (x_3 - X)^2 + (y_3 - Y)^2 - b^2 = 0; \\ (5) & x_2^2 + y_2^2 - K^2 = 0; & (6) & x_3^2 + y_3^2 - K^2 = 0; \end{array}$$

$$(7) \quad x_1^2 + y_1^2 - 2cx_1 = 0.$$

The result of eliminating $x_1, y_1, x_2, y_2, x_3, y_3$ gives an equation of the first degree, which establishes that the linkage will trace a straight line. There are various ways of effecting this elimination.

1. What element of the situation is left unused by the following procedure in the elimination?
   (*a*) From equations (1), (3), (5) eliminate x_2 and y_2 and obtain an equation
   $$(8) \quad f_1(x_1, y_1) = 0.$$
   (*b*) From equations (2), (4), (6) eliminate x_3 and y_3 and obtain an equation
   $$(9) \quad f_2(x_1, y_1) = 0.$$
   (*c*) From equations (7), (8), (9) eliminate x_1 and y_1 and obtain the desired equation.
2. How should this procedure be supplemented to secure the result?

[1] If reference is made to the article on "Linkages" in the December, 1915, MONTHLY, by Mr. Leavens, the following coördinates may be applied to his figure: $O(0, 0)$; $C(c, 0)$; $P_1(x_1, y_1)$; $M(x_2, y_2)$; $M_1(x_3, y_3)$; $P_2(X, Y)$.